Beechen Cliff School

Numeracy Booklet

A guide for pupils, parents and staff

Introduction

What is the purpose of the booklet?

This booklet has been produced to give guidance to pupils, parents and staff on how certain common Numeracy topics are taught in Mathematics classes and throughout the school. Staff from a wide range of departments have been consulted during its production. It is hoped that using a consistent approach across all subjects will make it easier for pupils to progress.

How can it be used?

You can use this booklet to help you solve Number, Measures and Handling Data problems in any subject. Look up the relevant page for a step by step guide.

If you are a pupil and your parents are helping you with your homework, they can refer to the booklet so they can see what methods you are being taught in school.

Why do some topics include more than one method?

In some cases (e.g. percentages), the method used will be dependent on the level of difficulty of the question, and whether or not a calculator is permitted.

For mental calculations, you should try to develop a variety of strategies so that you can use the most appropriate method in any given situation.

What if the topic I am looking for is not featured in this booklet?

As pupils progress through the school the use of much of this content becomes more rigorous and applied. If you are a pupil or a parent then looking through your Maths exercise book should provide examples to help you. Failing that, mymaths.co.uk is a comprehensive and easy to use website designed to walk you through similar problems. Additionally, feel free to contact any of the Maths teachers who will assist you with your problem.

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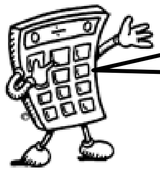


There are a number of useful mental strategies for addition. Some examples are given below.

Example Calculate 54 + 27

Method 1 Add tens, then add units, then add together

50 + 20 = 70   4 + 7 = 11   70 + 11 = 81

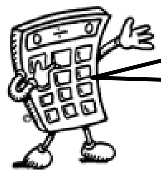
Method 2 Split up number to be added into tens and units and add separately.

54 + 20 = 74   74 + 7 = 81

Method 3 Round one (or both) of the numbers up to nearest 10, then subtract

54 + 30 = 84   but 30 is 3 too much so subtract 3;
84 - 3 = 81

Written Methods:

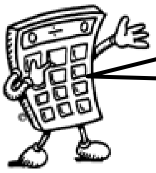


When adding numbers, ensure that the numbers are lined up according to place value. Start at right hand side, write down units, carry tens.

Example Add 3032 and 589

3032	3032	3032	3032
+589	+589	+589	+589
1	21	621	3621
1	1 1	1 1	1 1
2 + 9 = 11	3+8+1=12	0+5+1=6	3 + 0 = 3

Subtraction



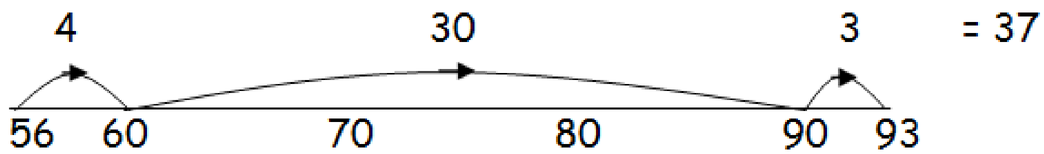
We use decomposition as a written method for subtraction (see below). **Alternative methods may be used for mental calculations.**

Mental Strategies:

Example Calculate 93 - 56

Method 1 Count on

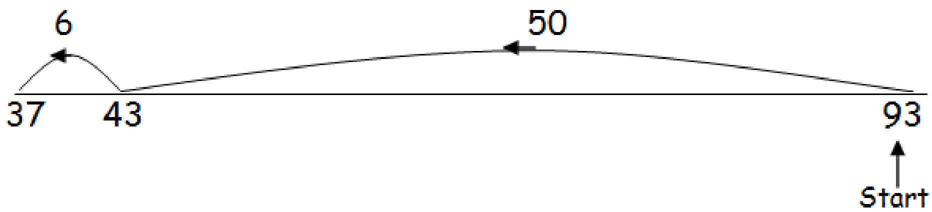
Count on from 56 until you reach 93. This can be done in several steps e.g.



Method 2 Break up the number being subtracted

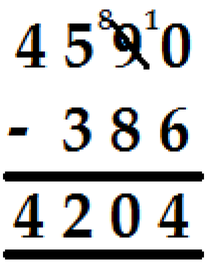
i.e. subtract 50, then subtract 6

93 - 50 = 43
43 - 6 = 37

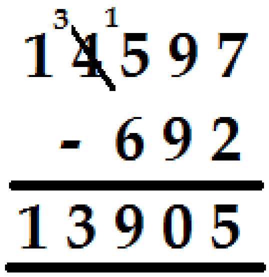


Written Methods:

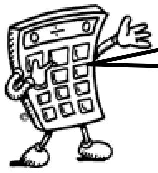
Example 1 4590 – 386



Example 2 Subtract 692 from 14597



Multiplication



It is essential that you know all of the multiplication tables from 1 to 12. These are shown in the “times tables” grid below.

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Mental Strategies:

Example Find 39 x 6

Method 1: “39 is 30 + 9 so we can multiply each by 6 and add the results together”

30 x 6
= 180

9 x 6
= 54

180 + 54
= 234

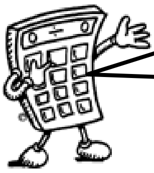
Method 2: “39 is close to 40, so we can multiply this by 6 and adjust at the end”

40 x 6
=240

40 is 1 too many
so take away 6x1

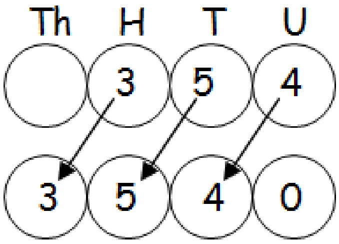
240 - 6
= 234

Multiplication 2



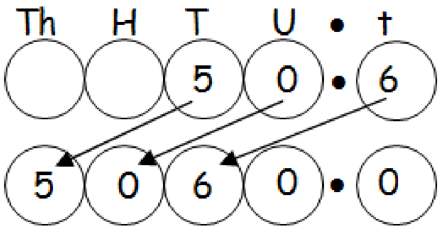
To multiply by **10** you move every digit ***one*** place to the left.
(add 0's on the end if necessary)
To multiply by **100** you move every digit ***two*** places to the left.
(add 0's on the end if necessary)

Example 1 (a) Multiply 354 by 10



So… 354 x 10 = 3540

(b) Multiply 50.6 by 100



So… 50.6 x 100 = 5060

(c) 35 x 30

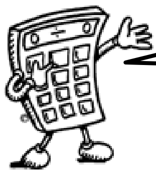
To multiply by 30, multiply by 3, then by 10

35 x 3 = 105
105 x 10 = 1050
So… 35 x 30 = 1050

(d) 436 x 600

To multiply by 600, multiply by 6, then by 100

436 x 6 = 2616
2616 x 100 = 261600
So… 436 x 600 = 261600



We may also use these rules for multiplying decimals numbers

Example 2 (a) 2.36 x 20

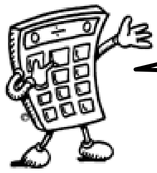
2.36 x 2 = 4.72
4.72 x 10 = 47.2
So… 2.36 x 20 = 47.2

b) 38.4 x 50

38.4 x 5 = 192.0
192.0x 10 = 1920
So… 38.4 x 50 = 1920

Multiplication 3

Long Multiplication:



We use long-multiplication when multiplying two numbers together that have two or more digits.

Example 1 Find 32 x 45

$$\begin{array}{r} 32 \\ \underline{\times\ 45} \\ 160 \\ \underline{+128\mathbf{0}} \\ 1440 \end{array}$$

Long Multiplication for 2 digits is done in three steps. First we multiply 32 by 5. Then we multiply 32 by 4 but because 4 is in the tens column we are actually multiplying by 40. Finally we add the two lines together to get the answer.

A zero is placed at the end of this row, because the 4 is worth 40.

So…. $\underline{32 \times 45 = 1440}$

Example 2 143 x 215

$$\begin{array}{r} 143 \\ \underline{\times\ 215} \\ 715 \\ 143\mathbf{0} \\ \underline{+286\mathbf{00}} \\ 30745 \end{array}$$

Multiplying 143 by 5

Multiplying 143 x 10 we put a zero at the end of the row.

Multiplying 143 x 200 we put 2 zeros at the end of the row

So…. $\underline{143 \times 215 = 30745}$

Example 3 2.1 x 3.2

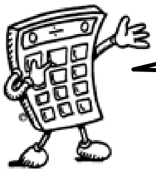
Treat as: 21 x 32

$$\begin{array}{r} 21 \\ \underline{\times\ 32} \\ 42 \\ \underline{+63\mathbf{0}} \\ 672 \end{array}$$

Now since there are 2 decimal places in the question we will expect 2 in the answer.

Alternatively estimate that 2.1 x 3.2 ≈ 2 x 3 = 6. So our answer of 672 must be corrected to be in the same order of magnitude as 6

So…. $\underline{2.1 \times 3.2 = 6.72}$



You should be able to divide by a single digit or by a multiple of 10 or 100 without a calculator.

Written Method:

Example 1 There are 192 pupils in first year, shared equally between 8 classes. How many pupils are in each class?

$$\begin{array}{r} 0\;2\;4 \\ 8\,\overline{\smash{)}\,1\,{}^{1}9\,{}^{3}2} \end{array}$$

8 goes into 1, no times remainder 1. This remainder is now carried over to make 19. Now 19 is divided by 8 which is 2 remainder 3. This remainder is carried over to make 32. 32 divided by 8 is 4.

There are 24 pupils in each class

Example 2 Divide 4.74 by 3

$$\begin{array}{r} 1\,.\,5\;8 \\ 3\,\overline{\smash{)}\,4\,.\,{}^{1}7\,{}^{2}4} \end{array}$$

When dividing a decimal number by a whole number, the decimal points must stay in line.

So….<u>4.74 ÷ 3 = 1.58</u>

Example 3 A jerry can contains 22 litres of water. If it is poured evenly into 80 bottles, how much water is in each bottle?

22 ÷ 80 = 22 ÷ (8 x 10) = 2.2 ÷ 8

Divide 22 by 10 first

$$\begin{array}{r} 0\,.\,2\;7\;5 \\ 8\,\overline{\smash{)}\,2\,.\,{}^{2}2\,{}^{6}0\,{}^{4}0} \end{array}$$

If you have a remainder at the end of a calculation, add a zero onto the end of the decimal and continue with the calculation.

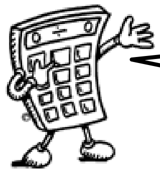
So…There are **<u>0.275 litres</u>** (275 ml) in each bottle

Order of Calculations (BIDMAS)

Consider this: What is the answer to 2 + 5 x 8 ?

Is it 7 x 8 = 56 or 2 + 40 = 42 ?

The correct answer is 42.



Calculations which have more than one operation need to be done in a particular order. The order can be remembered by using the mnemonic **BIDMAS.**

The **BIDMAS** rule tells us which operations should be done first.

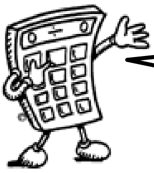
BIDMAS represents:

(B)rackets
(I)ndex
(D)ivide
(M)ultiply
(A)dd
(S)ubract

Scientific calculators use this rule, some basic calculators may not, so take care in their use.

Example 1	15 – 12 ÷ 6 = 15 - 2 = 13	BIDMAS tells us to divide first
Example 2	(9 + 5) x 6 = 14 x 6 = 84	BIDMAS tells us to work out the brackets first
Example 3	18 + 6 ÷ (5 - 2) = 18 + 6 ÷ 3 = 18 + 2 = 20	Brackets first Then divide Finally add
Example 4	5 – 3 + 4 = 2 + 4 = 6	<u>NOT</u> a BIDMAS problem Simply follow the operations from left to right

Evaluating Formulae



To find the value of a variable in a formula, we must substitute all of the given values into the formula, then use BIDMAS rules to work out the answer.

Example 1

Use the formula $P = 2L + 2B$ to evaluate P when $L = 12$ and $B = 7$.

$P = 2L + 2B$	Step 1: write formula
[P = 2xL + 2xB]	
$P = \quad 2 \text{ x } 12 + 2 \text{ x } 7$	Step 2: substitute numbers for letters
$P = \quad 24 \quad + \quad 14$	Step 3: start to evaluate (BIDMAS)
$P = \quad 38$	Step 4: write answer

Example 2

Use the formula $I = \dfrac{V}{R}$ to evaluate I when $V = 240$ and $R = 40$

$$I = \frac{V}{R}$$

$$I = \frac{240}{40}$$

$$I = 240 \div 40$$

$$I = 6$$

This is the formula that relates Current (I), Resistance (R) and Voltage (V)

Example 3

Use the formula $F = 32 + 1.8C$ to evaluate F when C = 20

$$F = 32 + 1.8C$$

$$F = 32 + 1.8 \text{ x } 20$$

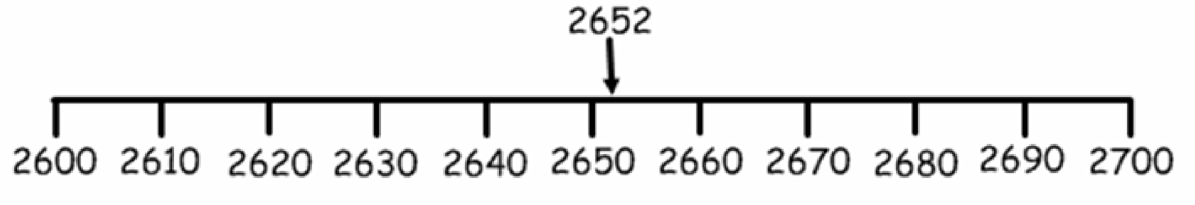
$$F = 32 + 36$$

$$F = 68$$

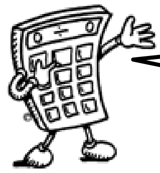
This is the formula used to convert temperature from Celsius(C) to Fahrenheit(F)
Therefore 20°C = 68°F

Estimation: Rounding

Numbers can be rounded to give an approximation.



26**5**2 rounded to the nearest 10 is 2650.
2**6**52 rounded to the nearest 100 is 2700.



When rounding numbers which are exactly in the middle, the set way is to round up.
78**6**5 rounded to the nearest 10 is 78**7**0.

The same principle applies to rounding decimal numbers.

In general, to round a number, we must first identify the place value to which we want to round. We must then look at the next digit to the right ("check digit")

If it is 5 or more round up, if not, leave it!

Example 1 Round 46,753 to the nearest thousand.

6 is the digit in the thousands column - the check digit (in the hundreds column) is a **7**, so round up.

4**6**753 = 47000 to the nearest thousand

Example 2 Round 1.57359 to 2 decimal places

The second number after the decimal point is a **7** - the check digit (the third number after the decimal point) is a **3**, so leave it! This is in effect rounds the number down.

1.5**7**359 = 1.57 to 2 decimal places

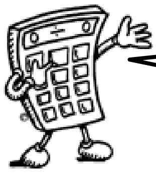
Example 3 Round 0.0403945 to 3 significant figures

4 is the first number of significance, **5** is the 3rd significant figure – the check digit is a **9** so round up

0.040**5**945 = 0.0406 to 3sf

Example 4 Round 63906 to 2 significant figures: 63**9**06 = 64000 to 2sf

Estimation: Calculation



We can use rounded numbers to give us an approximate answer to a calculation. This allows us to check that our answer is sensible.

Example 1

Tickets for a concert were sold over 4 days. The number of tickets sold each day was recorded in the table below. How many tickets were sold in total?

Monday	Tuesday	Wednesday	Thursday
486	205	197	321

Estimate = 500 + 200 + 200 + 300 = 1200

Calculate:

```
 486
 205
 197
+321
1209
```

Answer = 1209 tickets

Example 2

A bar of chocolate weighs 42g. There are 48 bars of chocolate in a box. What is the total weight of chocolate in the box?

Estimate = 40 x 50 = 2000g

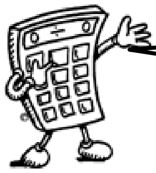
We normally try to round each part to 1 significant figure

Calculate:

```
  42
 x48
 336
1680
2016
```

Answer = 2016g

Time



Time may be expressed in 12 or 24 hour notation

12-hour clock

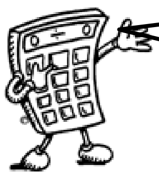
Time can be displayed on a clock face, or digital clock

These clocks both showfifteen minutes past five, or quarter past five.

When writing times in 12 hour clock, we need to add a.m. or p.m. after the time.

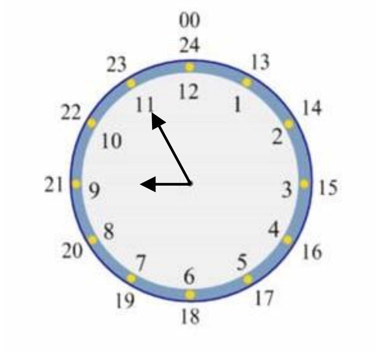
- a.m. is used for times between midnight and 12 noon (morning)
- p.m. is used for times between 12 noon and midnight (afternoon / evening).

24-hour clock



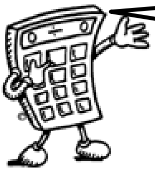
In 24 hour clock, the hours are written as numbers between 00 and 24. Midnight is expressed as 0000. After 12 noon, the hours and numbered 13, 14, 15 etc. We get these new numbers by adding 12 to the time, eg 6pm is 12 + 6 = 18

Examples



9.55 am	→	0955hrs
3.35 pm	→	1535hrs
12.20 am	→	0020hrs
0216 hrs	→	2.16 am
2045 hrs	→	8.45 pm

Time 2



It is essential to know the number of months, weeks and days in a year, and the number of days in each month.

Time Facts

In 1 year, there are:	365 days (366 in a leap year)
	52 weeks (add one day)
	12 months

The number of days in each month can be remembered using the rhyme:

"30 days hath September,
April, June and November,
All the rest have 31,
Except February alone,
Which has 28 days clear,
And 29 in each leap year."

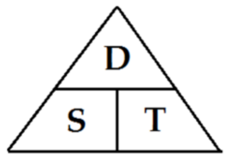
Distance, Speed and Time.

For any given journey, the distance travelled depends on the speed and the time taken. If speed is constant, then the following formulae apply:

Distance = Speed x Time or $D = S \times T$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad S = \frac{D}{T}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} \quad \text{or} \quad T = \frac{D}{S}$$



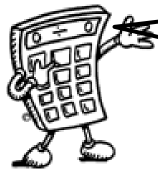
It may be useful to use this formula triangle to help you remember. Simply cover up the unit you need.

Example: Calculate the speed of a train which travelled 450 km in 5 hours.

$$S = \frac{D}{T}$$
$$S = \frac{450}{5}$$
$$S = 90km/h$$

NB: The units km per hr literally mean kms divided by hours, a distance divided a time. This reflects the formula for speed!

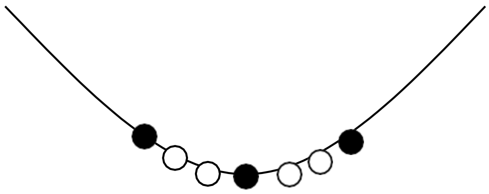
Fractions 1



Addition, subtraction, multiplication and division of fractions are studied in mathematics.
However, the examples below may be helpful in all subjects.

Understanding Fractions

Example
A necklace is made from black and white beads.



Question: What fraction of the beads are black?

Answer: There are 3 black beads out of a total of 7. So… $\frac{3}{7}$ of the beads are black.

Equivalent Fractions

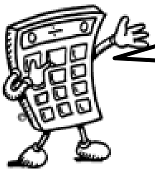
Example
What fraction of the flag is shaded?

6 out of 12 squares are shaded. So $\frac{6}{12}$ of the flag is shaded.

It could also be said that $\frac{1}{2}$ of the flag is shaded.

$\frac{6}{12}$ and $\frac{1}{2}$ are **equivalent fractions**

Fractions 2



The top of a fraction is called the **numerator**, the bottom is called the **denominator**.
To simplify a fraction, divide the **numerator** and **denominator** of the fraction by the same whole number

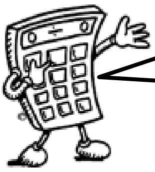
Example 1

a) $\frac{20}{25} \overset{\div 5}{\underset{\div 5}{=}} \frac{4}{5}$ b) $\frac{16}{24} \overset{\div 8}{\underset{\div 8}{=}} \frac{2}{3}$

This may need to be done repeatedly until the numerator and denominator are the smallest possible whole numbers - the fraction is then said to be in it's **simplest form**.

Example 2 Simplify $\frac{72}{84}$ Ans: $\frac{72}{84} = \frac{36}{42} = \frac{18}{21} = \frac{6}{7}$ (Simplest form)

<u>Calculating Fractions of a Quantity</u>



To find a fraction of a quantity, divide by the denominator and multiply by the numerator. If the numerator is 1, then it is only required to divide by the denominator

To find $\frac{1}{2}$ divide by 2, to find $\frac{1}{7}$ divide by 7 etc.

Example 1 Find $\frac{1}{5}$ of £150

$\frac{1}{5}$ of £150 = £150 ÷ 5 = £30

Example 2 Find $\frac{3}{4}$ of 48

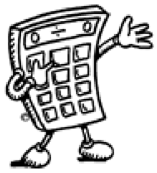
$\frac{1}{4}$ of 48 = 48 ÷ 4 = 12

$\frac{3}{4}$ of 48 = 12 x 3 = 36

To find $\frac{3}{4}$ of a quantity, divide by 4 then multiply by 3.

"<u>Divide by the bottom, multiply by the top</u>"

Percentages 1



Percent means **out of 100**.
A **percentage** can be converted to an equivalent **fraction** or **decimal.**

What is a Percentage?

36% means $\frac{36}{100}$ → 36% is therefore equivalent to $\frac{9}{25}$ (simplifying) and 0.36 (36 ÷ 100)

2% means $\frac{2}{100}$ → 2% is therefore equivalent to $\frac{1}{50}$ and 0.02 as a decimal.

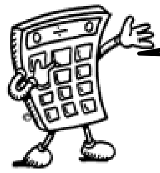
17.5 % means $\frac{17.5}{100}$ → 17.5% is therefore equivalent to $\frac{35}{200}$ and 0.175 as a decimal.

Common Percentages

Some percentages are used very frequently. It is useful to know these as fractions and decimals

Percentage	Fraction	Decimal
1%	$\frac{1}{100}$	0.01
10%	$\frac{10}{100}=\frac{1}{10}$	0.1
20%	$\frac{20}{100}=\frac{1}{5}$	0.2
25%	$\frac{25}{100}=\frac{1}{4}$	0.25
$33^1/_3$%	$\frac{1}{3}$	0.333…= $0.\dot{3}$
50%	$\frac{50}{100}=\frac{1}{2}$	0.5
$66^2/_3$%	$\frac{2}{3}$	0.666…= $0.\dot{6}$
75%	$\frac{75}{100}=\frac{3}{4}$	0.75

Percentages 2



There are many ways to calculate percentages of a quantity. Some of the common ways are shown below.

Non-Calculator Methods

Method 1 Using Equivalent Fractions

Example Find 25% of £640

25% of £640 = $\frac{1}{4}$ of £640 = £640 ÷ 4 = £160

Method 2 Using 1%

In this method, first find 1% of the quantity (by dividing by 100), then multiply to give the required value.

Example Find 9% of 200g

1% of 200g = 200g ÷ 100 = 2g

So 9% of 200g = 9 x 2g = <u>18g</u>

Method 3 Using other % amounts

This method is similar to the one above. Find amounts such as 10% (÷10), 50%(÷2), 25%(÷4) etc. Then add/multiply amounts to get the required percentage.

Example Find 43% of £360

10% of £360 = £360 ÷ 10 = £36

1% of £360 = £36 ÷ 10 = £3.60 (using the above)

43% = 4x10% + 3x1%

= 4 x £36 + 3 x £3.60

= £144 + £10.80

= £154.80

So… <u>43% of £360 is £154.80</u>

Percentages 3

Calculator Method 1

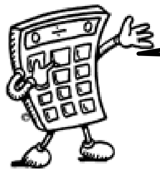
To find the percentage of a quantity using a calculator, change the percentage to a decimal, then multiply (called the **multiplier**)

Example Find 23% of £15000

23% = 0.23 (or $\frac{23}{100}$)

So 23% of £15000 = 0.23 x £15000 = £3450

(or 23% of £15000 = 23 ÷ 100 x 15000 = £3450)



Warning: We do not use the % button on calculators because some calculators work differently.

Calculator Method 2

This method is same as the non-calculator method for finding 1% first.
Divide the amount by 100, then multiply by the percentage required.

Example House prices increased by 19% over a one year period. What is the new value of a house which was valued at £236,000 at the start of the year?

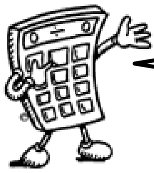
236000 ÷ 100 × 19 = 44840

Value at end of year = original value + increase
= £236000 + £44840
= £280840

The new value of the house is £280,840

Percentages 4

Finding the Percentage



To find one amount as a percentage of another, first make a fraction, then convert to a decimal by dividing the top by the bottom. Finally multiply by 100 to change from a decimal to a percentage.

Example 1 There are 30 pupils in Class 9A3. 18 Live in BA2. What percentage of Class 9A3 live in BA2

$\frac{18}{30}$ = 18 ÷ 30 = 0.6 (x100) = 60%

Therefore, 60% live in BA2

Example 2 James scored 36 out of 44 his biology test. What is his percentage mark?

Score = $\frac{36}{44}$ = 36 ÷ 44 = 0.81818… (x100)

= 81.818..%

= 82% (2sf)

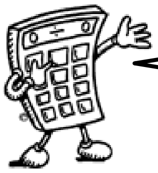
Example 3 In class 10A1, 14 pupils had brown hair, 6 pupils had blonde hair, 3 had black hair and 2 had red hair. What percentage of the pupils were blonde?

Total number of pupils = 14 + 6 + 3 + 2 = 25, 6 out of 25 were blonde, so,

$\frac{6}{25}$ = 6 ÷ 25 = 0.24 (x100) = 24%

Therefore, 24% were blonde

Ratio 1



When quantities are to be mixed together, the ratio, or proportion of each quantity is often given. The ratio can be used to calculate the amount of each quantity, or to share a total into parts.

Writing Ratios

Example 1:

To make a fruit drink, 4 parts water is mixed with 1 part of cordial.
The ratio of water to cordial is 4:1 (said “4 to 1”)
The ratio of cordial to water is 1:4.

Order is important when writing ratios

Example 2:

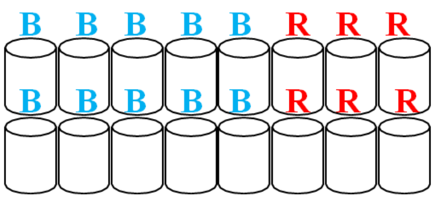
In a bag of balloons, there are 5 red, 7 blue and 8 green balloons.
The ratio of **red : blue : green** is therefore 5 : 7 : 8

Simplifying Ratios

Ratios can be simplified in much the same way as fractions

Example 1 Purple paint can be made by mixing 10 tins of blue paint with 6 tins of red. The ratio of blue to red can be written as 10 : 6

It can also be written as 5 : 3, as it is possible to split up the tins into 2 groups, each containing 5 tins of blue and 3 tins of red.



Blue : Red

$$\begin{array}{c} 10 : 6 \\ \div 2 \quad \div 2 \\ \underline{5 : 3} \end{array}$$

To simplify a ratio, divide each number in the ratio by a common factor.

Simplifying Ratios (continued)

Example 2

Simplify each ratio:

(a) 4 : 6 (b) 24 : 36 (c) 6 : 3 : 12

(a) 4 : 6
2 : 3

Divide each number by 2

(b) 24 : 36
2 : 3

Divide each number by 12

(c) 6 : 3 : 12
2 : 1 : 4

Divide each number by 3

Example 3 Concrete is made by mixing 20 kg of sand with 4 kg cement. Write the ratio of sand to cement in its simplest form.

Sand : Cement
20 : 4
5 : 1

Using ratios

The ratio of fruit to nuts in a chocolate bar is 3 : 2. If a bar contains 15g of fruit, what weight of nuts will it contain?

	Fruit	Nuts	
x5	3	2	x5
	15	**10**	

Multiply both sides of the ratio by the same number.

So the chocolate bar will contain 10g of nuts.

Ratio 3

Sharing in a given Ratio

Example: Lauren and Sean earn money by washing cars. By the end of the day they have made £90. As Lauren did more of the work, they decide to share the profits in the ratio 3 : 2. How much money did each receive?

Step 1 - **Add** together the numbers in the ratio to find the total number of parts

$$3 + 2 = 5$$

Step 2 - **Divide** the total by this number to find the value of each part

£90 ÷ 5 = £18 in each part

Step 3 - **Multiply** each figure by the value of each part

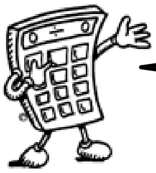
3 parts: 3 x £18 = £54
2 parts: 2 x £18 = £36

Step 4 - **Check** that the total is correct

£54 + £36 = £90 ✓

Lauren received £54 and Sean received £36

Proportion



Two quantities are said to be in direct proportion if when one doubles the other doubles and if one is halved the other is halved.

It is often useful to make a table when solving problems involving proportion.

Example 1 A car factory produces 1500 cars in 30 days. How many cars would they produce in 90 days?

Days	Cars
30	1500
90	**4500**

x3 x3

The factory would produce 4500 cars in 90 days.

Example 2 5 adult tickets for the cinema cost £27.50.
How much would 8 tickets cost?

Find the cost of one ticket first

	Tickets	Cost	
÷5	5	£27.50	÷5
	1	£5.50	
x8	8	£44.00	x8

Working:

$$\begin{array}{r} £5.50 \\ 5\,\overline{)\,£27.50} \end{array}$$

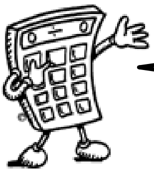
(Cost of one)

$$\begin{array}{r} £5.50 \\ \times 8 \\ \hline £44.00 \\ \hline \end{array}$$

4 4

The cost of 8 tickets is £44

Handling Information: Bar Charts



Bar graphs are used to display data that is in separate categories - **categorical data**. You must keep an equal gap between the bars.

Example: How do pupils travel to school?

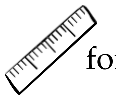
The table below shows the different methods pupils use to travel to school

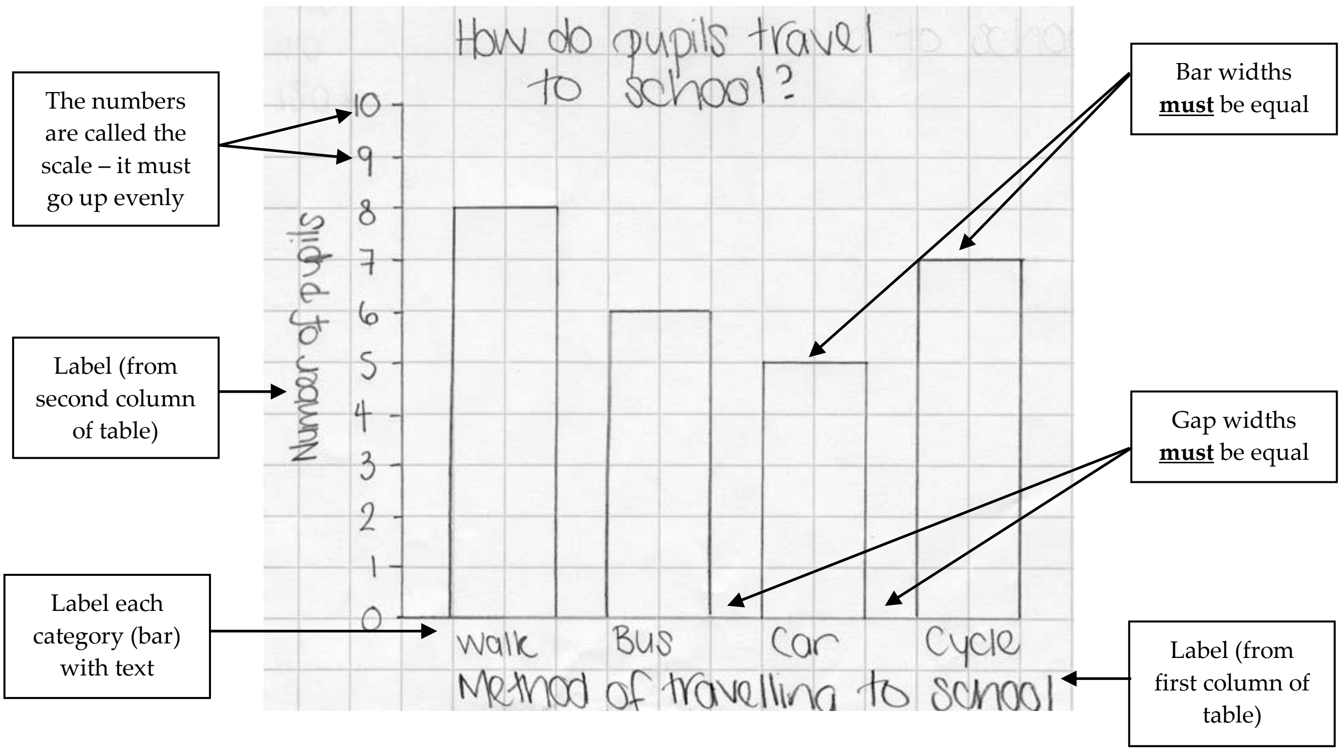
Method of travelling to school	Number of pupils
Walk	8
Car	6
Bus	5
Cycle	7

The first heading is the label for the horizontal axis (at the bottom of graph).

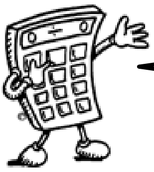
The second heading is the label for the vertical axis (at the side of the graph).

This data can be displayed as a bar graph

Remember: Use a pencil  and a ruler  for drawing your graph



Handling Information: Frequency Diagrams



Sometimes referred to as a Histogram; these look similar to Bar Charts and often used for **grouped numerical data**.

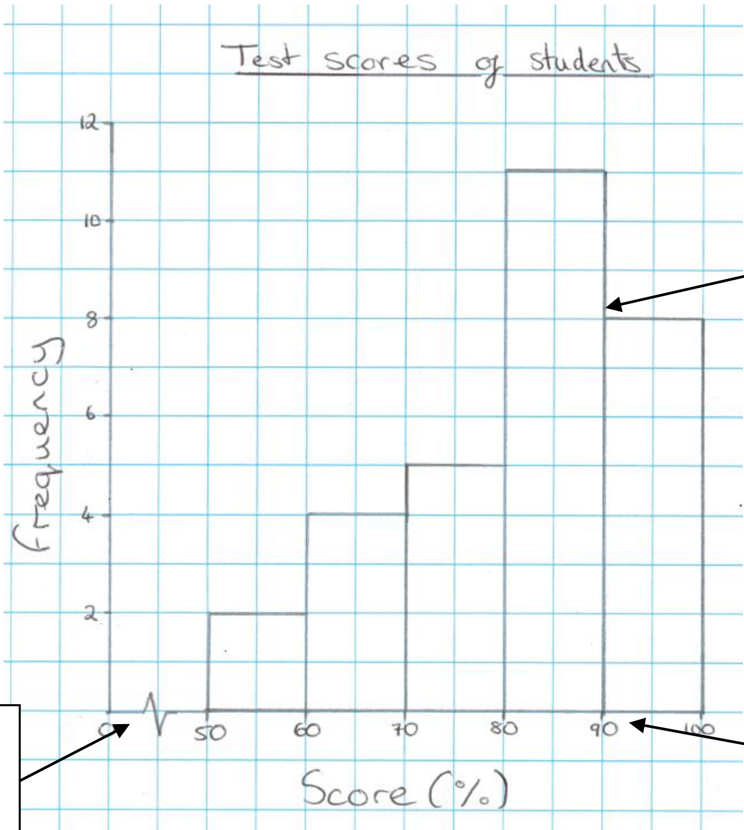
Example: Below are the results scored by a class of students.

Score (%)	Frequency
50 < score ≤ 60	2
60 < score ≤ 70	4
70 < score ≤ 80	5
80 < score ≤ 90	11
90 < score ≤ 100	8

This means greater than 50, but less than or equal to 60

This is the number of pupils in each interval

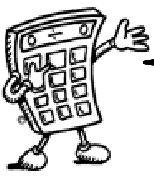
This can be displayed as a **grouped frequency chart** (histogram).



There are NO gaps between the bars – unless the frequency is zero

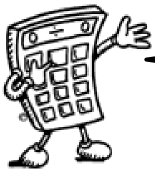
This symbol denotes a gap between 0 and 50, no data is in between these values

We use a **continuous scale** here (like a ruler). NOT the class labels from the frequency table



NB – Some histograms do not have to have equal class widths, in this case we work out the **frequency density**.

Handling Information: Line Graphs



Line graphs can help us spot trends. We can see how one thing changes as another changes. Usually how one thing changes with **time.**

Example: How does temperature affect dissolving?

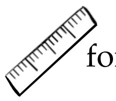
Tom performed an experiment to find out how changing the temperature of water affected the time for a vitamin C tablet to completely dissolve. His data is show in the table below

The first heading in the table is used to label the horizontal axis (remember to include the units).

Temperature of Water (°C)	Time to dissolve (seconds)
0	80
10	70
20	60
30	50
40	40

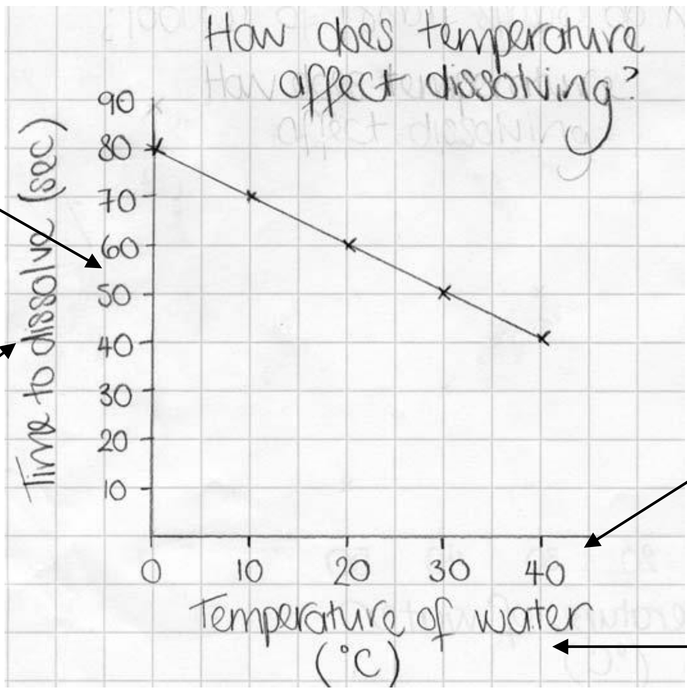
The second heading in the table is used to label the vertical axis (remember to include the units).

This data can be displayed as a line graph

Remember: Use a pencil  and a ruler  for drawing your graph

Scale on the vertical axis goes up evenly.

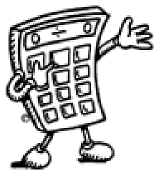
Label & units (from second column of table).



Scale on the horizontal axis goes up evenly.

Label & units (from first column of table).

Handling Information: Scatter Graphs

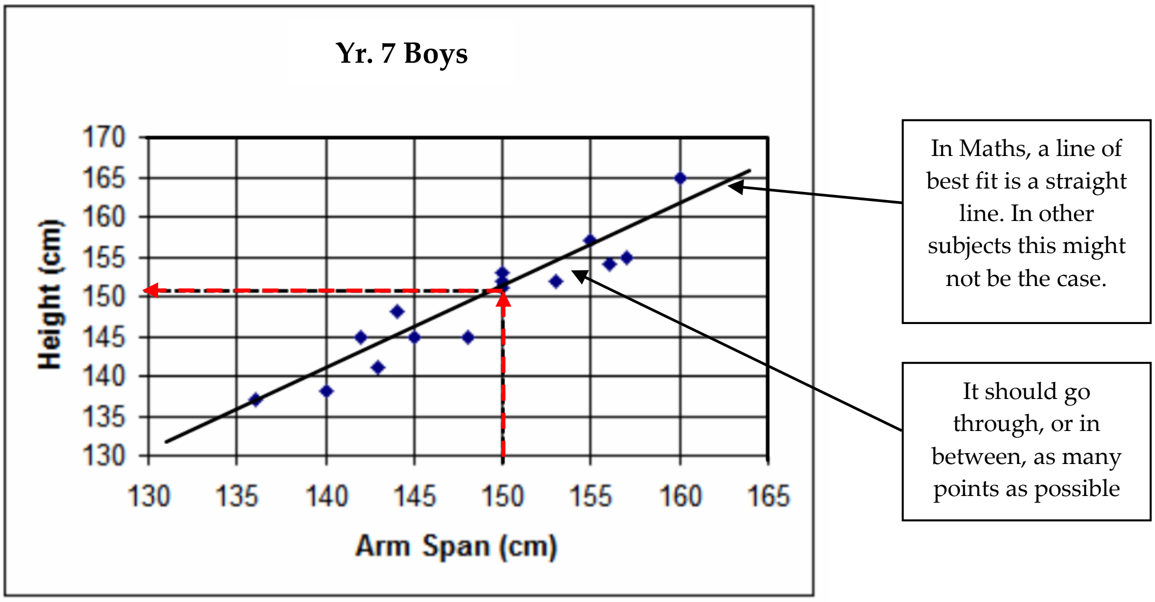


A scatter diagram is used to display the relationship between two variables. A pattern may appear on the graph. This is called a **correlation**.

Example: The table below shows the height and arm span of a group of year 7 boys. This is then plotted as a series of points on the graph below.

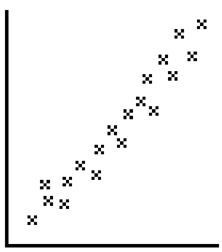
Arm Span (cm)	150	157	155	142	153	143	140	145	144	150	148	160	150	156	136
Height (cm)	153	155	157	145	152	141	138	145	148	151	1451	65	152	154	137

NB – In our table, Arm span is first so it is plotted on the horizontal axis and height is on the bottom so it is plotted on the vertical axis.

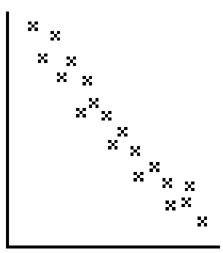


The line drawn is called the **line of best fit**. This line can be used to provide estimates. For example, a boy of arm span 150cm would be expected to have a height of around 151cm (shown by the dotted line drawn up and across the line of best fit)

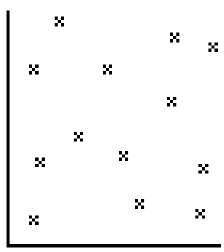
The graph shows a general trend, that as the arm span increases, so does the height. This graph shows a **positive correlation** (negative is a "downhill" line).



positive correlation

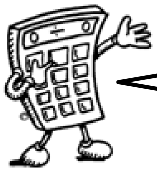


negative correlation



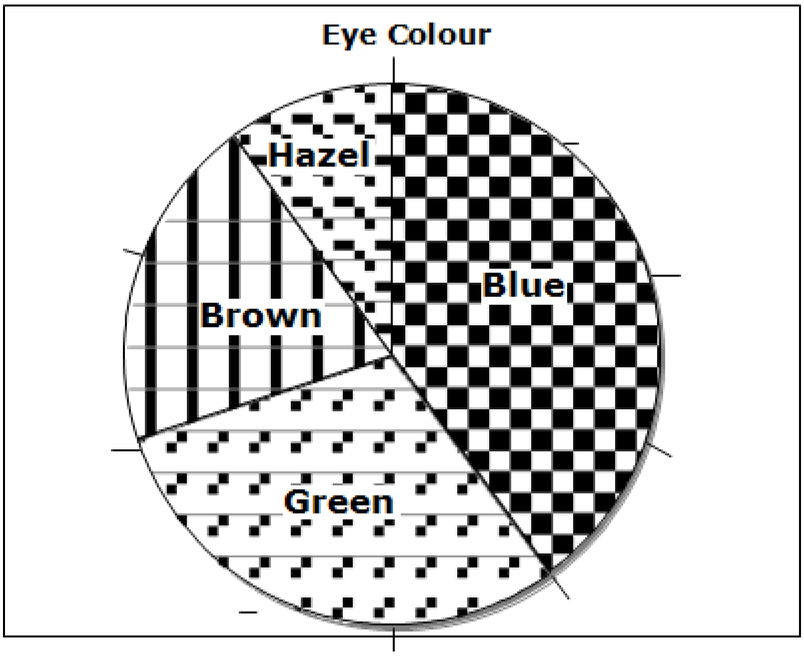
no correlation

Handling Information: Pie Charts



A pie chart can be used to display information. Each **sector** (slice) of the chart represents a different category. The size of each category can be worked out as a fraction of the total using the number of divisions or by measuring angles.

Example: 30 pupils were asked the colour of their eyes. The results are shown in the pie chart below.



The pie chart is divided up into ten parts, so each sector represents $\frac{1}{10}$ of the total.

To find the number of people for each sector

$$\text{Blue } \frac{4}{10} \text{ of } 30 = 30 \div 10 \times 4 = \underline{\underline{12}}$$

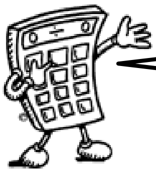
$$\text{Green } \frac{3}{10} \text{ of } 30 = 30 \div 10 \times 3 = \underline{\underline{9}}$$

$$\text{Brown } \frac{2}{10} \text{ of } 30 = 30 \div 10 \times 2 = \underline{\underline{6}}$$

$$\text{Hazel } \frac{1}{10} \text{ of } 30 = 30 \div 10 \times 1 = \underline{\underline{3}}$$

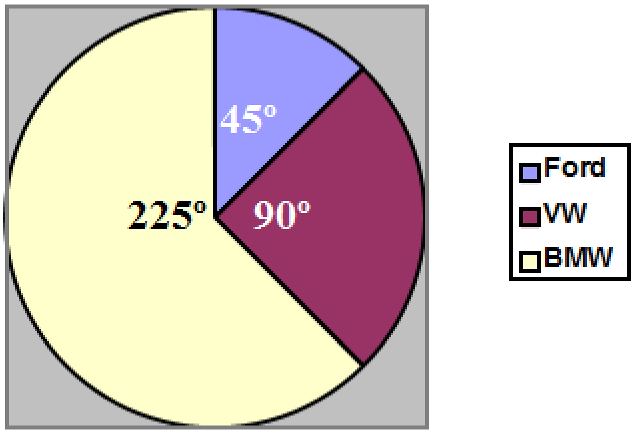
Reading Pie Charts with angles

Example The pie chart shows various makes of 600 cars sold in a showroom. Find the number of cars sold for each manufacturer.



If no divisions are marked, we can work out the fraction by measuring the angle for each sector.
Remember a **full circle** has **360°**

Make of Car



Ford: $\frac{45}{360} \times 600 = \underline{75 \text{ cars}}$

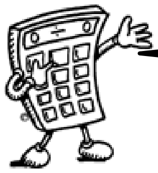
VW: $\frac{90}{360} \times 600 = \underline{150 \text{ cars}}$

BMW: $\frac{225}{360} \times 600 = \underline{375 \text{ cars}}$

(Check: 75 + 150 + 375 = 600)

Handling Information: Pie Charts 3

Drawing Pie Charts



On a pie chart, the size of the angle for each sector is calculated as a fraction of 360°

Example: In a survey about television programmes, a group of people were asked what their favourite TV show was. Their answers are given in the table below. Draw a pie chart to illustrate the information.

TV Show	Number of people
The Simpsons	28
The Voice	24
Britain's Got Talent (BGT)	10
The One Show	12
None	6

Total number of people = 80

The Simpsons (28): $\frac{28}{80} \rightarrow \frac{28}{80} \times 360^{\circ} = 126^{\circ}$

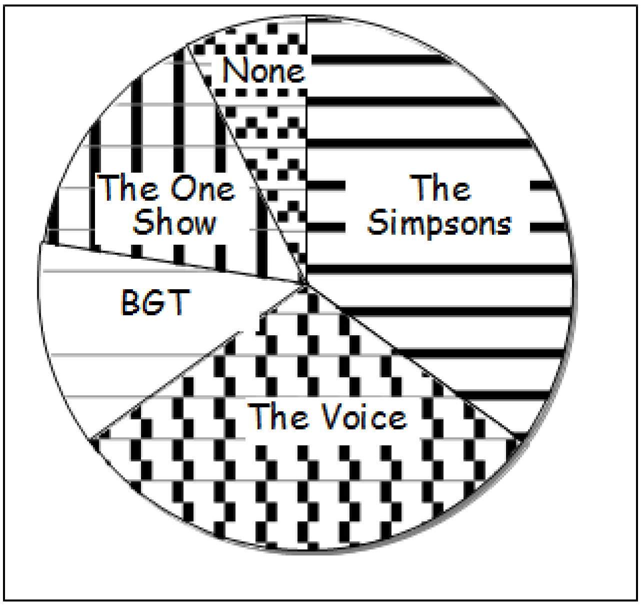
The Voice (24): $\frac{24}{80} \rightarrow \frac{24}{80} \times 360^{\circ} = 108^{\circ}$

BGT (10): $\frac{10}{80} \rightarrow \frac{10}{80} \times 360^{\circ} = 45^{\circ}$

The One Show (12): $\frac{12}{80} \rightarrow \frac{12}{80} \times 360^{\circ} = 54^{\circ}$

None(6): $\frac{6}{80} \rightarrow \frac{6}{80} \times 360^{\circ} = 27^{\circ}$

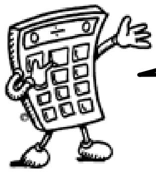
Favourite TV Show



Check that the total = 360°

Use a protractor to measure the angles when drawing a pie chart.

Handling Information: Averages



To provide information about a set of data, the average value may be given. There are **3 types** of **average** value – the **mean**, the **median** and the **mode**.

Mean - "average"
The mean is found by adding all the data together and dividing by the number of values.
Median - "middle"
The median is the middle value when all the data is written in numerical order (if there are two middle values, the median is half-way between these values).
Mode - "most common"
The mode is the value that occurs most often. NB: there can be more than one mode or even No mode!
Range
The range of a set of data is a measure of spread. NB: This is not officially an average

Range = Highest value – Lowest value

Example Class 9A2 scored the following marks for their homework. Find the mean, median, mode and range of the results.

7, 9, 8, 5, 6, 7, 10, 9, 8, 4, 8, 5, 6, 10

$$\text{Mean} = \frac{7+9+8+5+6+7+10+9+8+4+8+5+6+10}{14}$$

$$= \frac{102}{14} = 7.285...$$

Mean = 7.3 to 1 decimal place

Ordered values: 4, 5, 5, 6, 6, 7, $\boxed{7,\ 8}$, 8, 8, 9, 9, 10, 10

Median = 7.5

8 is the most frequent mark, so Mode = 8

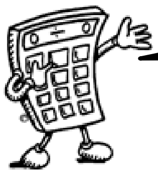
Range = 10 – 4 = 6

These are acceptable, even though they don't feature in the original data

Measurement

The metric system of measurement is used.

To measure length and distance we use **millimetres** (**mm**), **centimetres** (**cm**), **metres** (**m**) and **kilometres**(**km**).



In most subjects the most commonly used unit of measurement is millimetres.

To convert: 1cm = 10mm
1m = 100cm
1km = 1000m

Example 1: How many millimetres in 9.2cm?
9.2 x 10 = 92mm

Multiply by 10 to convert from centimetres to millimetres.

How centimetres in 3m?
3 x 100 = 300cm

Multiply by 100 to convert from metres to centimetres.

How many kilometres in 7000m?
7000 ÷ 1000 = 7km

Divide by 1000 to convert from metres to kilometres.

Area is measured in **square centimetres** (**cm²**).

To measure volume we use **cubic centimetres**(**cm³**) and liquid volumes are measured in **millilitres** (**ml**) and **litres** (**l**).

To convert $1cm^3 = 1ml$
1 litre = 1000ml = $1000cm^3$

Example 2 How many millilitres in 5.5 litres?
5.5 x 1000 = 5500ml

Multiply by a 1000 to convert from litres to millilitres.

How many litres in 3500ml?
3500 ÷ 1000 = 3.5 litres

Divide by 1000 to convert from millilitres to litres.

Mathematical Dictionary (Key Words)

Add; Addition (+)	To combine 2 or more numbers to get one number (called the sum or the total). Example: $12 + 76 = 88$
a.m.	(ante meridiem) Any time in the morning (between midnight and 12 noon).
Approximate	An estimated answer, often obtained by rounding to nearest 10, 100 or decimal place.
Calculate	Find the answer to a problem. It doesn't mean that you must use a calculator!
Data	A collection of information (may include facts, numbers or measurements).
Denominator	The bottom number in a fraction (the number of parts into which the whole is split).
Difference (-)	The amount between two numbers (subtraction). Example: The difference between 50 and 36 is 14 $50 - 36 = 14$
Division (÷)	Sharing a number into equal parts. $24 \div 6 = 4$
Double	Multiply by 2.
Equals (=)	Makes or has the same amount as.
Equivalent fractions	Fractions which have the same value. Example $\frac{6}{12}$ and $\frac{1}{2}$ are equivalent fractions.
Estimate	To make an approximate or rough answer, often by rounding.
Evaluate	To work out the answer.
Even	A number that is divisible by 2. Even numbers end with 0, 2, 4, 6 or 8.
Factor	A number which divides exactly into another number, leaving no remainder. Example: The factors of 15 are 1, 3, 5, 15.
Frequency	How often something happens. In a set of data, the number of times a number or category occurs.
Greater than (>)	Is bigger or more than. Example: 10 is greater than 6. $10 > 6$
Least	The lowest number in a group (minimum).
Less than (<)	Is smaller or lower than. Example: 15 is less than 21. $15 < 21$.

Maximum	The largest or highest number in a group.
Mean	The arithmetic average of a set of numbers (see p33).
Median	Another type of average - the middle number of an ordered set of data (see p33).
Minimum	The smallest or lowest number in a group.
Minus (-)	To subtract.
Mode	Another type of average – the most frequent number or category (see p33).
Most	The largest or highest number in a group (maximum).
Multiple	A number which can be divided by a particular number, leaving no remainder. Example Some of the multiples of 4 are 8, 16, 48, 72
Multiply (x)	To combine an amount a particular number of times. Example: 6 x 4 = 24
Negative Number	A number less than zero. Shown by a minus sign. Example: - 5 is a negative number.
Numerator	The top number in a fraction.
Odd Number	A number which is not divisible by 2. Odd numbers end in 1, 3, 5, 7 or 9.
Operations	The four basic operations are addition, subtraction, multiplication and division.
Order of operations	The order in which operations should be done. BIDMAS (see p10).
Place value	The value of a digit dependent on its place in the number. Example: in the number 1573.4, the 5 has a place value of 100.
p.m.	(post meridiem) Any time in the afternoon or evening (between 12 noon and midnight).
Prime Number	A number that has exactly two factors (can only be divided by itself and 1). NB: 1 is not a prime number as it only has one factor.
Product	The answer when two numbers are multiplied together. Example: The product of 5 and 4 is 20.
Range	The difference between the highest and the lowest value.
Remainder	The amount left over when dividing a number.
Share	To divide into equal groups.
Sum	The total of a group of numbers (found by adding).
Total	The sum of a group of numbers (found by adding).

Money Dictionary (Key words)

Account	A place to hold money in a bank or building society. Each account is given a unique number to identify this money; it is called the account number.
Budget	A specific amount of money to be spent on goods or services. For example Jane budgets £200 to spend on her holiday.
Credit	Money that is available to spend. It can be on a credit card, in the form of a bank loan or be money in a bank account.
Debit	Money that a person takes out of a bank account. If a person pays for goods with a debit card the money comes directly out of their account.
Debt	A sum of money that is owed or due
Deductions	The income tax and national insurance that an employer takes off a person's earnings.
Gross	The amount of money earned before deductions are made.
Income Tax	A tax collected directly from peoples earnings. The amount of money a person earns affects how much tax they pay. This tax pays for public services such as schools and hospitals.
Interest	Interest is provided at an agreed percentage rate. A person can be charged interest on a loan or mortgage and can earn interest on money in a bank account.
Loan	A sum of money that is expected to be paid back with interest
Loss	When you sell something for less than you paid for it
Mortgage	An amount of money loaned by a bank or building society to help buy a property. The mortgage is paid back monthly, with interest. Failure to pay can result in the house being repossessed.
National Insurance	A percentage of the money people earn that must be paid to the government. It pays for pensions, unemployment and sickness benefits.
Net	The amount of money earned a person can keep after the deductions have been made.
Profit	When a person sells something for more than they have paid for it.
Salary	The sum of money a person is paid over the course of a year.
VAT	Value added Tax. This is the tax paid when buying most goods. (Currently: 20%, 5% or 0%)

Notes

Feel free to use these blank pages for your own notes and calculations